

Weekly Review #11

Name: _____

1. The diagram shows triangle ABC. Point C has coordinates (4, 7) and the equation of the line AB is $x + 2y = 8$.

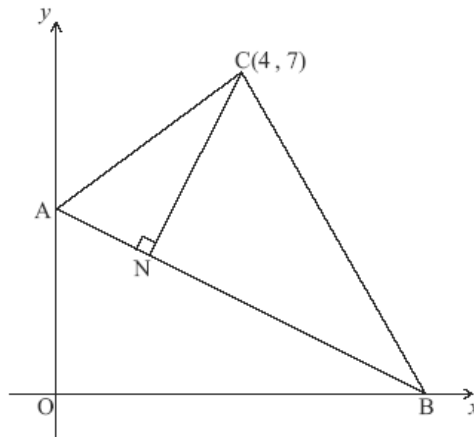


diagram not to scale

- (a) Find the coordinates of

(i) A;

(ii) B.

(2)

- (b) Show that the distance between A and B is 8.94 correct to 3 significant figures.

(2)

N lies on the line AB. The line CN is perpendicular to the line AB.

(c) Find

(i) the gradient of CN ;

(ii) the equation of CN.

(5)

(d) Calculate the coordinates of N.

(3)

It is known that $AC = 5$ and $BC = 8.06$.

(e) Calculate the size of angle ACB.

(3)

(f) Calculate the area of triangle ACB.

(3)

(Total 18 marks)

2. The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.

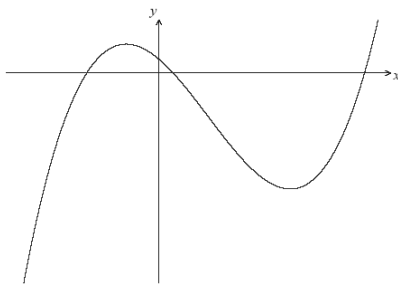


diagram not to scale

- (a) Write down the values of x where the graph of $f(x)$ intersects the x -axis. (3)
- (b) Write down $f'(x)$. (3)
- (c) Find the value of the local maximum of $y = f(x)$. (4)

Let P be the point where the graph of $f(x)$ intersects the y -axis.

- (d) Write down the coordinates of P. (1)
- (e) Find the gradient of the curve at P. (2)

The line, L , is the tangent to the graph of $f(x)$ at P .

- (f) Find the equation of L in the form $y = mx + c$.

(2)

There is a second point, Q , on the curve at which the tangent to $f(x)$ is parallel to L .

- (g) Write down the gradient of the tangent at Q .

(1)

- (h) Calculate the x -coordinate of Q .

(3)

(Total 19 marks)

3. Eight houses in a street are inhabited by different numbers of people, as shown in the table below.

House	A	B	C	D	E	F	G	H
Number of inhabitants	5	4	7	6	4	3	6	4

- (a) The following statements refer to the number of inhabitants per house.
Write down true (T) or false (F) for each.

(i) The mean is 5.

(ii) The range is 4.

(iii) The mode is 6.

(iv) The standard deviation is 1.4 correct to 2 significant figures.

(4)

- (b) Calculate the interquartile range for the number of inhabitants per house.

(2)

(Total 6 marks)

4. A questionnaire was given to all members of a school community to find out which drink was the most popular to have with breakfast. The results are given in the table below, classified by age.

	Hot Chocolate	Tea	Coffee	Milk
Children aged 12 years and less	55	10	1	34
Teenagers aged from 13 to 19 years	25	35	20	10
Adults aged 20 years and over	20	40	79	6

A χ^2 test was conducted to decide whether the type of drink was independent of age.

- (a) Find the number of degrees of freedom for the χ^2 test. (2)

- (b) Write down the null hypothesis for the χ^2 test. (1)

- (c) Write down the critical value for the χ^2 test at the 5 % significance level. (1)

The χ^2 test statistic is calculated to be 146 with a p -value of 6.62×10^{-29} (both numbers given correct to 3 significant figures).

- (d) Write down the conclusion reached at the 5 % significance level. Give a clear reason for your answer.

(2)
(Total 6 marks)

5. Consider $f: x \mapsto x^2 - 4$.

- (a) Find $f'(x)$. (1)

Let L be the line with equation $y = 3x + 2$.

- (b) Write down the gradient of a line parallel to L . (1)

- (c) Let P be a point on the curve of f . At P , the tangent to the curve is parallel to L . Find the coordinates of P .

(4)
(Total 6 marks)

6. Consider the function $f(x) = 3x + \frac{12}{x^2}$, $x \neq 0$.

(a) Differentiate $f(x)$ with respect to x .

(3)

(b) Calculate $f'(x)$ when $x = 1$.

(2)

(c) Use your answer to part (b) to decide whether the function, f , is increasing or decreasing at $x = 1$. Justify your answer.

(2)

(d) Solve the equation $f'(x) = 0$.

(3)

(e) The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P.

(i) Write down the coordinates of P.

(ii) Write down the gradient of T .

(iii) Write down the equation of T .

(5)

(f) Sketch the graph of the function f , for $-3 \leq x \leq 6$ and $-7 \leq y \leq 15$. Indicate clearly the point P and any intercepts of the curve with the axes.

(4)

- (g) (i) On your graph draw and label the tangent T .
- (ii) T intersects the graph of f at a second point. Write down the x -coordinate of this point of intersection.

(3)
(Total 22 marks)

7. Daniel wants to invest \$25 000 for a total of three years. There are three investment options.

Option One pays simple interest at an annual rate of interest of 6 %.

Option Two pays compound interest at a nominal annual rate of interest of 5 %, compounded **annually**.

Option Three pays compound interest at a nominal annual rate of interest of 4.8 %, compounded **monthly**.

- (a) Calculate the value of his investment at the end of the third year for each investment option, **correct to two decimal places**.

(8)

- (b) Determine Daniel's best investment option.

(1)
(Total 9 marks)

8. (a) Complete the truth table shown below.

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$(p \vee (p \wedge q)) \Rightarrow p$
T	T			
T	F			
F	T			
F	F			

(3)

- (b) State whether the compound proposition $(p \vee (p \wedge q)) \Rightarrow p$ is a contradiction, a tautology or neither.

(1)

Consider the following propositions.

p : Feng finishes his homework
 q : Feng goes to the football match

- (c) Write in symbolic form the following proposition.

If Feng does not go to the football match then Feng finishes his homework.

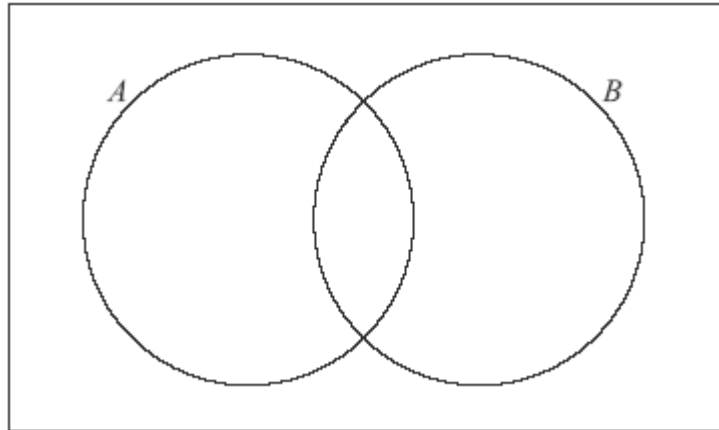
(2)

(Total 6 marks)

9. A group of 30 students were asked about their favourite topping for toast.

18 liked peanut butter (A)
10 liked jam (B)
6 liked neither

- (a) Show this information on the Venn diagram below.



(2)

- (b) Find the number of students who like both peanut butter and jam.

(2)

- (c) Find the probability that a randomly chosen student from the group likes peanut butter, given that they like jam.

(2)

(Total 6 marks)

10. The function $f(x) = 5 - 3(2^{-x})$ is defined for $x \geq 0$.

(a) (i) On the axes below sketch the graph of $f(x)$ and show the behaviour of the curve as x increases.

(ii) Write down the coordinates of any intercepts with the axes.



(4)

(b) Draw the line $y = 5$ on your sketch.

(1)

(c) Write down the number of solutions to the equation $f(x) = 5$.

(1)

(Total 6 marks)