What Came Before this Module:

In Grade 3, students explored area and discovered how area is related to multiplication and addition. In Grade 4, students constructed, recognized and defined geometric objects and used this knowledge to classify figures and solve problems. Students explored volume in Grade 5 using concrete models and developed the formula for volume of right rectangular prisms.

What Comes After this Module:

Students will use their knowledge of area, surface area and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms to solve real-world and mathematical problems in Grade 7. Students will also use the concepts from this module to compose and decompose figures into shapes they can work with to determine volume in Grade 7. In addition, students will use their knowledge of rates, ratios, and unit conversions to solve complex problems involving area and volume.

Area, Surface Area and Volume

Students utilize their previous experiences in shape composition and decomposition in order to understand and develop formulas for area, volume, and surface area.

How can you help at home?

- ✓ Every day, ask your child what they learned in school and ask them to show you an example.
- ✓ Ask your child to explain the relationship between a parallelogram and a rectangle.
- ✓ Ask your child to explain the relationship between a triangle and a rectangle.
- ✓ Challenge your child to determine possible dimensions for a right rectangular prism with a volume of 34 cubic centimeters. At least one measurement must be a fraction or decimal.

  (One possible solution: $6\frac{4}{5}$ cm x $2\frac{1}{2}$ cm x 2 cm)

Key Common Core Standards:

Solve real-world and mathematical problems involving area, surface area, and volume.

- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\,w\,h$ and $V = b\,h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Area of Parallelograms

Calculate the area of each parallelogram. The figures are not drawn to scale.
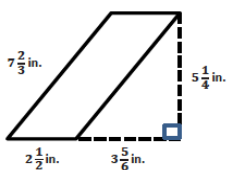
1.

Solution:

$A = bh$
$= 13\,cm(6\,cm)$
$= 78\,cm^2$

2.
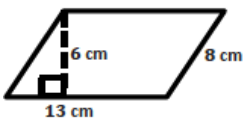
Solution:

$A = bh$
$= 2\frac{1}{2}\,in.\left(5\frac{1}{4}\,in.\right)$
$= \frac{5}{2}\,in.\left(\frac{21}{4}\,in.\right)$
$= \frac{105}{8}\,in^2$
$= 13\frac{1}{8}\,in^2$

Key Words

Triangular Region
A *triangular region* is the union of the triangle and its interior.

Altitude and Base of a Triangle
An *altitude* of a triangle is a perpendicular segment from a vertex of a triangle to the line containing the opposite side. The opposite side is called the *base*. For every triangle, there are three choices for the altitude, and hence there are three base-altitude pairs. The *height* of a triangle is the length of the altitude. The length of the base is called either the *base length* or more commonly, the *base*. Usually, context makes it clear whether the *base* refers to a number or a segment. These terms can mislead students: base suggests the bottom, while *height* usually refers to vertical distances. Do not reinforce these impressions by consistently displaying all triangles with horizontal bases. The altitude of the triangle in the image is shown by a dashed line.

Pentagon
Given 5 different points A, B, C, D, E in the plane, a 5-*sided polygon*, or *pentagon*, is the union of five segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EA}$ such that (1) the segments intersect only at their endpoints, and (2) no two adjacent segments are collinear. An example of a regula pentagon is shown to the right.

Hexagon
Given 6 different points A, B, C, D, E, F _in the plane, a 6-*sided polygon*, or *hexagon*, is the union of six segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EF}, \overline{FA}$ such that (1) the segments intersect only at their endpoints, and (2) no two adjacent segments are collinear. For both pentagons and hexagons, the segments are called the *sides*, and their endpoints are called the *vertices*. Like quadrilaterals, pentagons and hexagons can be denoted by the order of vertices defining the segments. For example, the pentagon *ABCDE* has vertices A, B, C, D, E that define the five segments in the definition above. Similar to quadrilaterals, pentagons and hexagons also have *interiors*, which can be described using pictures in elementary school. An example of a hexagon is shown on the right.

Line Perpendicular to a Plane
A line L intersecting a plane E at a point P is said to be *perpendicular to the plane E* if L is perpendicular to every line that (1) lies in E and (2) passes through the point P. A segment is said to be perpendicular to a plane if the line that contains the segment is perpendicular to the plane. In Grade 6, a line perpendicular to a plane can be described using a picture.

Parallel Planes
Two planes are *parallel* if they do not intersect. In Euclidean geometry, a useful test for checking whether two planes are parallel is if the planes are different and if there is a line that is perpendicular to both planes.

Right Rectangular Prism
Let E and E' be two parallel planes. Let B be a rectangular region 4 in the plane E. At each point P of B, consider the segment PP' perpendicular to E, joining P to a point P' of the plane E'. The union of all these segments is called a *right rectangular prism*. It can be shown that the region B' in E' corresponding to the region B is also a rectangular region whose sides are equal in length to the corresponding sides of B. The regions B and B' are called the *base faces* (or just *bases*) of the prism. It can also be shown that the planar region between two corresponding sides of the bases is also a rectangular region called the *lateral face* of the prism. In all, the boundary of a right rectangular prism has 6 *faces*: the 2 base faces and 4 lateral faces. All adjacent faces intersect along segments called *edges* (base edges and lateral edges). Here is an example of a right rectangular prism.

Cube
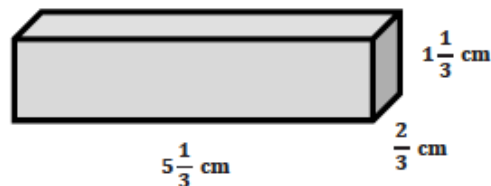A *cube* is a right rectangular prism all of whose edges are of equal length. Here is an example of a cube.

Surface of a Prism
The *surface of a prism* is the union of all of its faces (the base faces and lateral faces).

Finding the Volume of Right Rectangular Prisms with Fractional Side Lengths

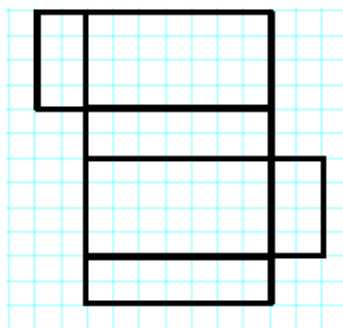1. Calculate the volume of the prism below.

Solution:

$$V = l\,w\,h$$

$$V = \left(5\frac{1}{3}\ cm\right)\left(\frac{2}{3}\ cm\right)\left(1\frac{1}{3}\ cm\right)$$

$$V = \frac{16}{3}\ cm \times \frac{2}{3}\ cm \times \frac{4}{3}\ cm$$

$$V = \frac{128}{27}\ cm^3$$

2. If you have to fill the prism with cubes whose side lengths are less than 1 cm, what size would be best?

Solution:

The best choice would be a cube with side lengths of $\frac{1}{3}$ cm.

Calculate the surface area of the rectangular prism. Assume each box in the grid represents a 1ft x 1ft square.

Solution:

$$(2ft\ x\ 4ft) + (2ft\ x\ 4ft) + (4ft\ x\ 7ft) + (4ft\ x\ 7\ ft) + (7ft\ x\ 2\ ft) + (7\ ft\ x\ 2ft) =$$

$$2(2\ ft\ x\ 4ft) + 2(4\ ft\ x\ 7\ ft) + 2(7ft\ x\ 2ft) =$$

$$16\ ft^2 + 56ft^2 + 28ft^2 = 100ft^2$$